About the Equilibrium Constant (K)
(Molecular biology of the cell page 159)

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Consider 1000 molecules of A and 1000 molecules of B. The concentrations of both are about 10^{-9} M. If the equilibrium constant (K) for A + B <--> AB is 10^{10} then one can calculate at equilibrium there will be: 270 A molecules, 270 B molecules and 730 AB molecules.
If equilibrium constant is 10^{8} which represents a loss of 2.8 kcal/mol of binding energy there will be: 915 A molecules, 915 B molecules and 85 AB molecules.
Note: this is from a figure in the text trying to explain the equilibrium constant not an actual question.
My problem: where are all of these values coming from? How do you determine the number of molecules present when given only the starting number of molecules and their concentrations? Any insight would help, I'm totally confused. THANKS!

Well it seems to be somekind of a trap. A Gifted genius says something as if it were quite simple, and the less gifted are confused not to understand anything. Natural selection makes the rest : sorrow and anger push the less Gifted ones out of Existence, and the way stands clear for the Gifted ones to multiply.

First you have to realize a little table representing the reaction A + b →AB
In the first line you put the concentration before reaction [A] =[B]=10^{-9}mol*l^{-1}
In the second you represent the concentrations at the equilibrium. Some part of A and B has evolved to give AB. Let's represent this part as x.

	A	B	AB
Reactants Before reaction	10^{-9} $\mathrm{mol} * \mathrm{l}^{-1}$	10^{-9} $\mathrm{mol} * \mathrm{l}^{-1}$	0
Products At equilibrium	10^{-9}-x	10^{-9} - x	x

Then you have to apply the equation of the equilibrium with the concentration of the products:

$$\frac{[\mathrm{AB}]}{[\mathrm{A}] * [\mathrm{B}]} = Ka = 10^{10}$$

In other words :

$$\frac{[\mathrm{x}]}{[10^{-9} - \mathrm{x}] * [\ 10^{-9} - \mathrm{x}]} = Ka = 10^{10}$$

But it's the same as :

$$[x] = Ka * [10^{-9} - \mathrm{x}] * [10^{-9} - \mathrm{x}]$$

$$[x] = Ka * [\ 10^{-9} * 10^{-9} - \mathrm{x} * 10^{-9} - \mathrm{x} * 10^{-9} + \mathrm{x}^2]$$

$$[x] = Ka * [10^{-18} - 2 * \mathrm{x} * 10^{-9} + \mathrm{x}^2]$$

$$Ka * [10^{-18} - 2 * \mathrm{x} * 10^{-9} + \mathrm{x}^2] - [x] = 0$$

$$Ka * [10^{-18} - 2 * \mathrm{x} * 10^{-9} + \mathrm{x}^2] - [x] = 0$$

$$Ka * [10^{-18} - 2 * x * 10^{-9} + x^2] - [x] * \frac{Ka}{Ka} = 0$$

$$Ka * \left[10^{-18} - 2 * x * 10^{-9} - \frac{[x]}{Ka} + x^2\right] = 0$$

$$Ka * \left[10^{-18} - x\left(2 * 10^{-9} - \frac{1}{Ka}\right) + x^2\right] = 0$$

$$Ka * [10^{-18} - x\,(2 * 10^{-9} - 1 * 10^{-10}) + x^2] = 0$$

$$Ka * [x^2 - x\,(2.1 * 10^{-9} + 10^{-18})] = 0$$

Let’s solve the equation :

$$[x^2 - x\,(2.1 * 10^{-9}) + 10^{-18}] = 0$$

$$\Delta = b^2 - 4ac = (2.1 * 10^{-9})^2 - 4 * 10^{-18} * (1)$$

$$\Delta = 4.41 * 10^{-18} - 4 * 10^{-18}$$

$$\Delta = 0.41 * 10^{-18}$$

$$\sqrt{0.41 * 10^{-18}} = 6.4 * 10^{-10}$$

Then we search for the roots

$$x1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2.1 * 10^{-9} - 6.4 * 10^{-10}}{2} = 7.29 * 10.^{-10}$$

$$x2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2.1 * 10^{-9} + 6.4 * 10^{-10}}{2} = 1.37 * 10.^{-9}$$

The second one is incorrect since it represents even more than the initial concentration. Therefore we must take the first solution.

$$x1 = 7.29 * 10.^{-10}$$

We know that at equilibrium :

$$[A] = 10^{-9} - x$$

Therefore

$$[A] = 10^{-9} - 7.29 * 10.^{-10}$$

And

$$[A] = 2.7 * 10^{-10} \text{mol} * l^{-1}$$

But we have not finished yet, there’s a little more gifted’s stuff :

At the beginning we had 1000 molecules of A an 1000 molecules of B. This number was said to be as much as $10^{-9}\text{mol} * l^{-1}$
But if 1000 molecules are as much as $10^{-9}\text{mol} * l^{-1}$,
100 are as much as $10^{-10}\text{mol} * l^{-1}$
200 are as much as $2 * 10^{-10}\text{mol} * l^{-1}$

270 are as much as $2.7 * 10^{-10} \mathrm{mol} * \mathrm{l}^{-1}$

Therefore we understand how the gifted one who wrote the article pulls out 270 molecules of his hat for A and B (without of course any explanation since he was writing apparently for other as gifted ones who would understand the trick in a flickering of an eyelid)

	[A]	[B]	[AB]
Reactants Before reaction	10^{-9} $\mathrm{mol} * \mathrm{l}^{-1}$	10^{-9} $\mathrm{mol} * \mathrm{l}^{-1}$	
Products At equilibrium	$2.7 * 10^{-10} \mathrm{mol} * \mathrm{l}^{-1}$	$2.7 * 10^{-10} \mathrm{mol} * \mathrm{l}^{-1}$	x$7.3 * 10^{-10} \mathrm{mol} * \mathrm{l}^{-1}$